Sesión Especial 10

EDP'S y aplicaciones/PDE'S and applications

Organizadores

- Begoña Barrios (Universidad de La Laguna)
- María Medina (Universidad de Granada)

Descripción

El objetivo de esta sesión es reunir a importantes especialistas, así como a prometedores jóvenes matemáticos en el área de ecuaciones en derivadas parciales para presentar los recientes progresos en dicho campo, identificar problemas abiertos y fomentar nuevas colaboraciones científicas entre distintos grupos.

Programa

MARTES, 5 de febrero (mañana)

11:30 – 12:10	Juan Luis Vázquez (Universidad Autónoma de Madrid) *Nonlocal elliptic equations of fractional type.*
12:10 – 12:50	José C. Sabina de Lis (Universidad de La Laguna) *On the spectrum of the p–Laplacian as p goes to* 1.
12:50 – 13:30	Xavier Ros-Oton (Universität Zürich) *On the regularity of the free boundary in the thin obstacle problem.*

MARTES, 5 de febrero (tarde)

17:00 – 17:40	Diana Stan (Universidad de Cantabria) *Carleman estimates for fractional operators.*
17:40 – 18:20	José M. Mazón (Universidad de Valencia) *El operador Dirichlet-Neumann asociado al* 1-*Laplaciano.*

JUEVES, 7 de febrero (mañana)

11:30 – 12:10	Ireneo Peral Alonso (Universidad Autónoma de Madrid) *The KPZ equation with fractional diffusion.*
12:10 – 12:50	Mariel Sáez Trumper (Pontificia Universidad Católica de Chile) *On the uniqueness of graphical mean curvature flow.*
12:50 – 13:30	Fernando Quirós (Universidad Autónoma de Madrid) *Logarithmic corrections in Fisher-KPP problems for the Porous Medium Equation.*

JUEVES, 7 de febrero (tarde)

15:30 – 16:10	Rafael López Soriano (Universidad de Valencia) *A mean field problem for conformal metrics with assigned curvatures.*
16:10 – 16:50	Cristina Brändle (Universidad Carlos III de Madrid) *Dynamical behaviour of populations living in separated domains interacting through the boundary.*
17:30 – 18:10	David Ruiz (Universidad de Granada) *Prescribing Gaussian curvature on compact surfaces and geodesic curvature on its boundary.*

VIERNES, 8 de febrero (mañana)

9:00 – 9:40	Angela Pistoia (La Sapienza Universita di Roma) *Maximal solution of the Liouville equation in doubly connected domains.*
9:40 – 10:20	Eduardo Colorado (Universidad Carlos III de Madrid) *Fractional semilinear elliptic equations with Dirichlet-Neumann boundary conditions.*
10:20 – 11:00	Matteo Bonforte (Universidad Autónoma de Madrid) *Nonlinear and nonlocal degenerate diffusions on bounded domains.*
11:30 – 12:10	Félix del Teso (BCAM) *The Liouville Theorem for Nonlocal Diffusion Operators (and its relation to irrational numbers and subgroups of $\mathbb{R}^N$).*
12:10 – 12:50	Fernando Soria (Universidad Autónoma de Madrid) *Inequalities related to PDE with fractional diffusion revisited.*

Dynamical behaviour of populations living in separated domains interacting through the boundary

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Abstract.
We consider two populations living separately in two different regions of the plane. The interaction between the populations occurs across the common boundary in the sense that only the individuals living "outside" move to the "inside" region, where they stay and become part of the inside-community.
We analyse the dynamics of the populations, by studying the asymptotic behaviour, the stationary states of the system and the stability of the solutions. A crucial issue in getting this dynamical behaviour is the asymptotic analysis of the linear associated problem when a parameter goes to infinity. It turns out that the existence and uniqueness of solutions as well as its long time behaviour depend on the geometrical distribution of some potentials, related to the heterogeneous character of the nonlinear problem.

Joint work with Pablo Álvarez Caudevilla.
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Nonlinear and Nonlocal Degenerate Diffusions on Bounded Domains

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Abstract. We study quantitative properties of nonnegative solutions to a nonlinear and nonlocal diffusion equation of the form $u_t = \mathcal{L}F(u)$ posed in a bounded domain, with appropriate homogeneous Dirichlet boundary conditions. The diffusion is driven by a linear operator $\mathcal{L}$ in a quite general class, that includes the three most common versions of the fractional Laplacians on a bounded domain with zero Dirichlet boundary conditions, as well as many other examples. The nonlinearity F is allowed to be degenerate, the prototype being $F(u) = |u|^{m-1}u$, with $m > 1$.
We will shortly present some recent results about existence, uniqueness and a priori estimates for a quite large class of very weak solutions, that we call weak dual solutions. Then we will concentrate on the regularity theory: decay and positivity, boundary behavior, Harnack inequalities, interior and boundary regularity, and asymptotic behavior. All this is done in a quantitative way, based on sharp a priori estimates. Although our focus is on the fractional models, our techniques cover also the local case s = 1 and provide new results even in this setting. A surprising instance of this problem is the possible presence of nonmatching powers for the boundary behavior: this unexpected phenomenon is a completely new feature of the nonlocal nonlinear structure of this model, and it is not present in the semilinear elliptic case, for which we will shortly present the most recent results.
The above results are contained on a series of recent papers in collaboration with A. Figalli, Y. Sire, X. Ros-Oton and J. L. Vazquez.

Fractional semilinear elliptic equations with Dirichlet-Neumann boundary conditions

Eduardo Colorado

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Abstract. In this talk we will show how mixed Dirichlet-Neumann boundary conditions affect to existence issues for non-linear elliptic problems involving the fractional Laplacian operator,

$$\left\{\begin{array}{rcll} (-\Delta)^s u & = & f(x,u) & \text{in } \Omega \\ u & = & 0 & \text{on } \Sigma_{\mathcal{D}} \\ \dfrac{\partial u}{\partial \nu} & = & 0 & \text{on } \Sigma_{\mathcal{N}}, \end{array}\right.$$

where f is a function satisfying certain regularity and growth properties, $\Omega \subset \mathbb{R}^N$ is a regular bounded domain, $\frac{1}{2} < s < 1$, ν is the outwards normal to $\partial\Omega$, $\Sigma_{\mathcal{D}}$, $\Sigma_{\mathcal{N}}$ are smooth $(N-1)$-dimensional submanifolds of $\partial\Omega$ such that $\Sigma_{\mathcal{D}} \cup \Sigma_{\mathcal{N}} = \partial\Omega$, $\Sigma_{\mathcal{D}} \cap \Sigma_{\mathcal{N}} = \emptyset$, and $\Sigma_{\mathcal{D}} \cap \overline{\Sigma}_{\mathcal{N}} = \Gamma$ is a smooth $(N-2)$-dimensional submanifold of $\partial\Omega$.

The Liouville Theorem for Nonlocal Diffusion Operators (and its relation to irrational numbers and subgroups of $\mathbb{R}^N$).

Félix del Teso

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Abstract. The classical Liouville Theorem states that

$$u \in L^\infty(\mathbb{R}^N) \quad \& \quad \Delta u = 0 \text{ in } \mathbb{R}^N \qquad \Longleftrightarrow u \equiv C \in \mathbb{R}.$$

In this talk we will revisit this result for the following class of nonlocal operators

$$\mathcal{L}^\mu[\psi](x) = \text{P.V.} \int_{|z|>0} (\psi(x+z) - \psi(x))\, \mathrm{d}\mu(z),$$

where μ is any positive symmetric Radon measure such that

$$\int_{\mathbb{R}^N} \text{mín}\{|z|^2, 1\} \mathrm{d}\mu(z) < +\infty.$$

This class of operators naturally includes the fractional Laplacian $(-\Delta)^s$ for $s \in (0,1)$, Relativistic Schrodinger operators $(-\Delta + m^2)^s - m^{2s}$, convolution operators $(J * \psi)(x) - \psi(x)$ as well as discretizations of both local and nonlocal symmetric diffusion operators.
First, we will treat the one dimensional case. Here we give a precise classification, in terms of the measure μ, for which the Liouville Theorem holds. The condition will be related to *irrational numbers* ([1]).
In $\mathbb{R}^N$ such a characterization is also proved. This time it will be given in terms of the *group* generated by the support of the measure μ ([1]).
This nonlocal result will allow us ([2]) to give a full characterization of the Liouville property for any operator (local + nonlocal, and not necessarily symmetric) with constant coefficients satisfying the maximum principle (see [3]), i.e

$$\underbrace{\text{Tr}(\sigma\sigma^T D^2\psi(x)) + b \cdot \nabla\psi}_{\text{Local}} + \underbrace{\int_{|z|>0} \left(\psi(x+z) - \psi(x) - z \cdot \nabla\psi(x) \mathbf{1}_{|z|\le 1}\right) \mathrm{d}\nu(z)}_{\text{Nonlocal}}.$$

Referencias

[1] N. Alibaud, F. del Teso, J. Endal, and E. R. Jakobsen. *Characterization of nonlocal diffusion operators satisfying the Liouville theorem. Irrational numbers and subgroups of* $\mathbb{R}^d$. Preprint: arXiv:1807.01843.

[2] N. Alibaud, F. del Teso, J. Endal, and E. R. Jakobsen. *The Liouville theorem and linear operators satisfying the maximum principle. A complete characterization in the constant coefficient case.* Work in progress.

[3] P. Courrège. *Générateur infinitésimal d'un semi-groupe de convolution sur R^n, et formule de Lévy-Khinchine.* Bull. Sci. Math. (2), 88:3–30, 1964.

Joint work with *N. Alibaud* (University of Besançon), *J. Endal* and *E. R. Jakobsen* (Norwegian University of Science and Technology).

A mean field problem for conformal metrics with assigned curvatures

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Abstract. This talk is concerned with a Liouville mean field problem on compact surfaces. The interest of this type of equations is motivated by some problems which arise in Differential Geometry and Physics.
We will focus on the case of surfaces with boundary and we will impose a particular Neumann boundary condition. This allows us to prescribe Gaussian and geodesic curvatures by conformal changes of the metric. This case has not been much considered in the literature and the results are partial. We give some results on the solvability of the problem using variational techniques, including a new Moser-Trudinger type inequality. In order to do it, we deal with the possible blow–up phenomenon and we establish a general criterion.

Joint work with Luca Battaglia (Università di Roma Tre)

El operador Dirichlet-Neumann asociado al 1-Laplaciano

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Resumen. El operador Dirichlet-Neumann asociado al p-Laplaciano con $p > 1$ ha sido amplia mente estudiado, sobre todo por su conexión con el problema inverso de Calderón. Nuestro objetivo es estudiar el operador Dirichlet-Neumann asociado al 1-Laplaciano. Una de las dificultades es su correcta definición, debido a la no unicidad del problema de Dirichlet

$$\begin{cases} -\Delta_1 u = 0 \quad \text{in } \Omega \\ \\ u = \varphi \quad \text{on } \partial\Omega. \end{cases}$$

Damos la correcta definición de dicho operador en $L^1(\partial\Omega)$ y demostramos que el problema de Cauchy asociado a dicho operador tiene una única solución fuerte para cada dato inicial en $L^1(\partial\Omega)$. Como consecuencia obtenemos que el problema elíptico con condiciones dinámicas de frontera

$$\begin{cases} -\Delta_1 u = \quad \text{in } (0,\infty) \times \Omega \\ \\ w_t + \frac{Du}{|Du|} \cdot \eta = g \quad \text{on } (0,\infty) \times \partial\Omega \\ \\ u = w \quad \text{on } (0,\infty) \times \partial\Omega \\ \\ w(0,x) = w_0 \quad x \in \partial\Omega, \end{cases}$$

tiene una única solución fuerte para cada dato inicial $w_0 \in L^1(\partial\Omega)$

Trabajo en colaboración con D. Hauer (Sydney University).

The KPZ equation with fractional diffusion

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Abstract. The classical Kardar-Parisi-Zang model involves a nonlinearity depending on the modulus of the gradient, but the diffusion is typically local, the laplacian, porous media, etc.

The goal of the talk is to explore a direction in which the diffusion is driven by a fractional Laplacian.

Referencias

[1] B. Abdellaoui, I. Peral, *Towards a deterministic KPZ equation with fractional diffusion: The stationary problem.* Nonlinearity 31 (2018) 1260-1298

[2] B. Abdellaoui, I. Peral, A. Primo, *On the KPZ equation with fractional diffusion.* Preliminary preprint.

Joint work with B. Abdellaoui and A. Primo. Supported by Ministerio de Economia y Competitividad under grants MTM2013-40846-P and MTM2016-80474-P.

Maximal solution of the Liouville equation in doubly connected domains

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Abstract. We prove the existence of solutions to the classical Liouville equation which blows-up along a smooth, simple and closed curve inside a doubly connected domain.

The result has been obtained in collaboration with Giusi Vaira and Mike Kowalczyk.

Logarithmic corrections in Fisher-KPP problems for the Porous Medium Equation

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Abstract. We consider the large time behaviour of solutions to the Porous Medium Equation with a Fisher-KPP type reaction term

$$u_t = \Delta u^m + u - u^2 \quad \text{in } \mathbb{R}^N \times \mathbb{R}_+, \qquad u(\cdot, 0) = u_0 \quad \text{in } \mathbb{R}^N,$$

$m > 1$, for nonnegative, nontrivial, radially symmetric, bounded and compactly supported initial data u_0. It is well known that in spatial dimension one there is a minimal speed $c_* > 0$ for which the equation admits a traveling wave profile Φ_{c_*} with a finite front. We prove that there exists a second constant $c^* > 0$ independent of the dimension N and the initial function u_0, such that

$$\lim_{t\to\infty} \left\{ \sup_{x\in\mathbb{R}^N} |u(x,t) - \Phi_{c_*}(|x| - c_* t + (N-1)c^* \log t - r_0)| \right\} = 0$$

for some $r_0 \in \mathbb{R}$ (depending on u_0). Moreover, the radius, $h(t)$, of the support of the solution at time t satisfies

$$\lim_{t\to\infty} \left[h(t) - c_* t + (N-1)c^* \log t \right] = r_0.$$

Thus, in contrast with the semilinear case $m = 1$, we have a logarithmic correction only for $N > 1$. If the initial function is not radially symmetric, there exist $r_1, r_2 \in \mathbb{R}$ such that the boundary of the spatial support of the solution $u(\cdot, t)$ is contained in the spherical shell $\{x \in \mathbb{R}^N : r_1 \le |x| - c_* t + (N-1)c^* \log t \le r_2\}$ for all $t \ge 1$. Moreover, as $t \to \infty$, $u(x,t)$ converges to 1 uniformly in $\{|x| \le c_* t - (N-1)c \log t\}$ for any $c > c^*$.

Joint work with Yihong Du and Maolin Zhou.

On the regularity of the free boundary in the thin obstacle problem

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Abstract. The thin obstacle problem is a classical free boundary problem arising in a variety of settings: elasticity, semi-permeable membranes, optimal stopping, etc. From the mathematical point of view, the most challenging question in this context is to understand the regularity of the free boundary. It is known that the free boundary is smooth outside a certain set of degenerate points (near which the solution grows "less than expected"). In full generality, however, nothing can be said about such set of degenerate points, and they could form even a large fractal set. Thus, it is an important question to determine conditions on the given obstacle and/or on the boundary data under which the set of degenerate points behaves nicely. The aim of this talk is to present two different results in this direction, one of them in collaboration with B. Barrios and A. Figalli, the other one in collaboration with X. Fernández-Real.

Referencias

[1] B. Barrios, A. Figalli, X. Ros-Oton, *Global regularity for the free boundary in the obstacle problem for the fractional Laplacian*, Amer. J. Math. 140 (2018), 415-447.

[2] X. Fernández-Real, X. Ros-Oton, in preparation.

Joint works with Begoña Barrios and Alessio Figalli, and with Xavier Fernandez-Real.

Prescribing Gaussian curvature on compact surfaces and geodesic curvature on its boundary

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Abstract. The problem of prescribing the Gaussian curvature on compact surfaces via a conformal change of the metric dates back to the works of Berger, Moser, Kazdan-Warner, etc. Our aim is to consider surfaces with boundary where we also prescribe the geodesic curvature of the border. This gives rise to a Liouville equation under nonlinear Neumann boundary conditions.
In this talk we address the case of negative Gaussian curvature, and we will focus on the blow-up analysis of the solutions. Here the cancellation between the area and length terms make it possible to have blowing-up solutions with unbounded total mass. This phenomenon seems to be entirely new in the related literature. We are able to give a complete description of this question under Morse index restrictions.

This is joint work with Andrea Malchiodi (SNS Pisa) and Rafael López Soriano (U. Valencia).

On the spectrum of the p–Laplacian as p goes to 1

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Resumen.
Se presentan algunos resultados sobre el límite de los autovalores del problema

$$\begin{cases} -\Delta_p u = \lambda |u|^{p-2} u & x \in \Omega \\ u = 0 & x \in \partial\Omega, \end{cases}$$

cuando $p \to 1$. Se supone que $\Omega \subset \mathbb{R}^N$ es un dominio acotado de clase $C^{0,1}$ mientras los autovalores λ se observan en el sentido de Ljusternik–Schnirelman. Bajo condiciones favorables se describe asimismo el perfil asintótico de las autofunciones.

Referencias

[1] K.C. Chang, *The spectrum of the 1–Laplace operator*. Commun. Contemp. Math. **11** (2009), no. 5, 865–894.

[2] J. Sabina de Lis, S. Segura de León, *On the limit as $p \to 1$ of the higher eigenvalues of the p–Laplacian*. Trabajo en preparación.

Trabajo en colaboración con Sergio Segura de León (Universidad de Valencia).
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On the uniqueness of graphical mean curvature flow

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Abstract.
In this talk I will discuss recent work with P. Daskalopoulos on sufficient conditions to prove uniqueness of complete graphs evolving by mean curvature flow. It is interesting to remark that the behaviour of solutions to mean curvature flow differs from the heat equation, where non-uniqueness may occur even for smooth initial conditions if the behaviour at infinity is not prescribed for all times.

Trabajo en colaboración con Panagiota Daskalopoulos.
Financiado por Proyecto Fondecyt 1150014.

Inequalities related to PDE with fractional diffusion revisited

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Abstract. In this talk we'll review classical functional inequalities, like Hardy's inequality, weighted Sobolev inequalities and others, associated with the fractional Laplacian. Even though the results are not new, the proofs we provide come from different arguments that maybe of some interest for future applications.

Joint work with Ireneo Peral.

Carleman estimates for fractional operators

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Abstract. We consider the heat equation for fractional relativistic operators. Several properties of the operator are recalled and new ones are presented, including construction of eigenfunctions and fractional Leibniz rule. We prove backward uniqueness for the evolution equation. Moreover, we show several techniques useful to derive so called Carleman estimates: this includes monotonicity inequalities and convexity of certain energy functionals.

Joint work with Luz Roncal and Luis Vega
Partially supported by grant Juan de la Cierva FJCI-2015-25797, Severo Ochoa excellence grant SEV-2013-0323 and ERCEA Advanced Grant 2014 669689-HADE.

Nonlocal elliptic equations of fractional type

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Abstract. The talk presents a work on the existence and behavior of solutions of nonlinear fractional elliptic equations, mainly when posed in bounded domains.

Trabajo en colaboración con Ildefonso Díaz y David Gómez Castro, UCM
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